April 26, 2019 Homework 8 due May 9, 2019

1. Let let G be a group. If for each $a \in G$ one has $a^2 = 1$, then for all $a, b \in G$ one has $ab = ba$.

2. Let $G = \mathbf{R} \setminus \{-1\}$. Define $a * b = a + b + ab$. Show that $(G, *)$ is a group.

3. Let $y_a a = a$ and $y_b b = b$. Show that $y_a = y_b$.

4. Show that the set $\{f_{m,b} : \mathbf{R} \to \mathbf{R} : m \neq 0, \text{ and } f_{m,b}(x) = mx + b\}$ forms a group under composition of functions.

5. Show that the set of matrices $\begin{bmatrix} m & b \\ 0 & 1 \end{bmatrix}$ with $m \neq 0$ is a group under matrix multiplication.

6. Let G be a group, and $|G| = 2n$. True or False? There is $a \in G$ so that $a \neq 1$, and $a^2 = 1$.

7. Let S be a subset of $GL_2(\mathbf{R})$ that consists of symmetric matrices. True or False? S is a subgroup of $GL_2(\mathbf{R})$.

8. Let $a = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}$. Show that $a^n = \begin{bmatrix} f_{n+1} & -f_n \\ -f_n & f_{n-1} \end{bmatrix}$, where $f_0 = 0$, $f_1 = 1$, and $f_{n+1} = f_n + f_{n-1}$ is the Fibonacci sequence.