Homework 1 Solutions - Dr. Kogan's 221 to Graded Problems

1.1 #1, 3, 12, 17, 21, 23(a)   1.2 # 10, 12, 14, 20, 31

Section 1.1

1. $\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right] \xrightarrow{R_2 = 2R_1 + R_2} \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right] \xrightarrow{R_2 = \frac{1}{3}R_2} \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1 = R_1 - 5R_2}$

$\rightarrow \left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right]$ $x_1 = -8,\ x_2 = 3$ or $(x_1, x_2) = (-8, 3)$

$-2 + 6$

12. $\left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ 0 & -6 & 10 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -6 & 10 & -3 \end{array}\right] \rightarrow$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{array}\right] \leftarrow$ The system is <u>inconsistent</u> because this row is in the form $0x_1 + 0x_2 + 0x_3 = 5$. This cannot be solved - it is false.

21. $\left[\begin{array}{cc|c} 1 & 4 & -2 \\ 3 & h & -6 \end{array}\right]$ consistent if no rows are of the form $0x_1 + 0x_2 =$ a nonzero value

$\downarrow$

$\left[\begin{array}{cc|c} 1 & 4 & -2 \\ 0 & h-12 & 0 \end{array}\right]$ Any value of h will produce a consistent system.

If $h = 12$, the system is consistent <u>and</u> has infinite solutions.

23. a. True. All elementary row operations are reversible. See page 6 of textbook for more info.

Section 1.2

10. $\left[\begin{array}{ccc|c} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 4 \\ 0 & 0 & -7 & 14 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right] \rightarrow$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$

Gives the equations: $x_1 - 2x_2 = 2$

$x_3 = -2$

So, $\begin{cases} x_1 = 2 + 2x_2 \\ x_2 = \text{free} \\ x_3 = -2 \end{cases}$

12. $\left[\begin{array}{cccc|c} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$ → Inconsistent System (no solution) because row is of form $0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$ which has no solution.

14. $\left[\begin{array}{ccccc|c} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \underbrace{\left[\begin{array}{ccccc|c} 1 & 0 & -5 & 0 & 0 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]}_{\text{reduced echelon form}}$ $\begin{cases} x_1 = -5x_3 + 3 \\ x_2 = 6 - 4x_3 + x_4 \\ x_3 = \text{free} \\ x_4 = \text{free} \\ x_5 = 0 \end{cases}$

20. $\left[\begin{array}{cc|c} 1 & -3 & 1 \\ 2 & h & k \end{array}\right]$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & -3 & 1 \\ 0 & h+6 & k-2 \end{array}\right]$

a) No solution
$h = -6, \; k \neq 2$

b) Unique Solution
$h \neq -6$

c) Infinitely Many Solutions
$h = -6, \; k = 2$