MATH221
quiz #1, 02/27/20
Solutions
Total 100

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Show all work legibly. **Name**:______________________

1. (40) This problem consists of two parts dealing with systems of linear equation $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = 0$.

   (a) (20) Solve the system

$$\begin{array}{rrrrrcl} x_1 & & -x_3 & & -2x_5 & = & 1 \\ & x_2 & +3x_3 & & -x_5 & = & 2 \\ 2x_1 & & -2x_3 & +x_4 & -3x_5 & = & 0 \end{array}$$

Solution.

$$\begin{bmatrix} 1 & 0 & -1 & 0 & -2 & 1 \\ 0 & 1 & 3 & 0 & -1 & 2 \\ 2 & 0 & -2 & 1 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 & -2 & 1 \\ 0 & 1 & 3 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

   (b) (20) Consider the homogeneous system of linear equations

$$\begin{array}{rrrrrcl} x_1 & & -x_3 & & -2x_5 & = & 0 \\ & x_2 & +3x_3 & & -x_5 & = & 0 \\ 2x_1 & & -2x_3 & +x_4 & -3x_5 & = & 0 \end{array}$$

Identify vectors that span the solution set of the system.

Solution. The vectors are (see solution to part (a)):

$$\begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \text{ and } \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

2. (20) Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a set of linearly independent vectors, and $\mathbf{w}_1 = \mathbf{u}_1$, $\mathbf{w}_2 = \mathbf{u}_2$, $\mathbf{w}_3 = \mathbf{u}_3 + \mathbf{u}_1$. True or False? The vectors $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ are linearly independent.

   Solution. If $c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + c_3\mathbf{w}_3 = 0$, then

   $$0 = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3(\mathbf{u}_1 + \mathbf{u}_3) = (c_1 + c_3)\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3.$$

   This yields $c_1 + c_3 = 0$, $c_2 = 0$, $c_3 = 0$, and $c_1 = c_2 = c_3 = 0$.

   Mark one and explain.

   ▫ True ▫ False

3. (20) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation that reflects a vector with respect to the (x, y) plane, and then rotates it 90^o clockwise. Find the standard matrix A of T.

   Solution.

   $$A = [T(\mathbf{e}_1), T(\mathbf{e}_2), T(\mathbf{e}_3)] = [-\mathbf{e}_2, \mathbf{e}_1, -\mathbf{e}_3] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

4. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

   (a) (15) Find vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ so that $A\mathbf{b}_1 = \mathbf{e}_1$, and $A\mathbf{b}_2 = \mathbf{e}_2$.

   Solution.

   $$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{bmatrix}.$$

   Finally

   $$\mathbf{b}_1 = \begin{bmatrix} -2 \\ 3/2 \end{bmatrix}, \text{ and } \mathbf{b}_2 = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}.$$

   $\mathbf{b}_1 = \begin{bmatrix} \quad \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} \quad \end{bmatrix}$

(b) (5) Let $B = [\mathbf{b}_1, \mathbf{b}_2]$. Compute $B\mathbf{a}_1$ and $B\mathbf{a}_2$.

Solution.

$$B\mathbf{a}_1 = \mathbf{e}_1, \text{ and } B\mathbf{a}_2 = \mathbf{e}_2.$$

$B\mathbf{a}_1 = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$ and $B\mathbf{a}_2 = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

5. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3] = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. True or False? The columns of A are linearly independent.

Solution. $\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 = 0$.

Mark one and explain.

□ True □ False