MATH221

quiz #2, 03/24/20

Solutions

Total 100

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly. **Name**:______________________

1. (20) Let $A = \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$, and $B = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$. Compute AB.

   $AB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.

2. (40) Let $A = \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$.

   (a) (20) Find A^{-1} if exists.

   Solution.

$$\begin{bmatrix} 2 & -2 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 3 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3/2 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

   $A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$.

   (b) (20) Solve the matrix system of equations $AX = B$ where $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

   Solution. $X = A^{-1}B = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.

3. (20) Let A be an invertible matrix. True or False? If $\{A\mathbf{u}_1, \ldots, A\mathbf{u}_n\}$ is a linearly independent set, then the vector set $\{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ is linearly independent.

   Solution. Let $c_1\mathbf{u}_1 + \ldots + c_n\mathbf{u}_n = 0$. Then

$$0 = A(c_1\mathbf{u}_1 + \ldots + c_n\mathbf{u}_n) = c_1A\mathbf{u}_1 + \ldots + c_nA\mathbf{u}_n, \text{ and } c_1 = \ldots = c_n = 0.$$

   Mark one and explain.

   ▫ True ▫ False

4. (20) Let $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$ be a 2×3 matrix. True or False? If columns of A are linearly independent, then the system $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$.

   Solution. Let $A_2 = [\mathbf{a}_1, \mathbf{a}_2]$. Since columns of the 2×2 matrix A_2 are linearly independent the system $A_2\mathbf{y} = \mathbf{b}$ has a solution $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = A_2^{-1}\mathbf{b}$ for each $\mathbf{b}$. That is $y_1\mathbf{a}_1 + y_2\mathbf{a}_2 = \mathbf{b}$. Clearly $y_1\mathbf{a}_1 + y_2\mathbf{a}_2 + 0\mathbf{a}_3 = \mathbf{b}$,

   One can also note that 3 vectors in $\mathbf{R}^2$ may NOT be linearly independent.

   Mark one and explain.
   □ True □ False

5. (20) Let A be an $n \times n$ invertible matrix. True or False? If B is an $n \times n$ matrix, and AB is invertible, then B is invertible.

   Solution. $B = \left(A^{-1}\right)(AB)$ is a product of two invertible matrices, hence B is invertible.

   Mark one and explain.
   □ True □ False