MATH411
quiz 0
02/04/2020
Total 100

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly. **Name**:______________________

1. (30) Let $\mathbf{u}_1$, $\mathbf{u}_2$ be linearly independent vectors of magnitude 1 (i.e., $\mathbf{u}_1^T\mathbf{u}_1 = \mathbf{u}_2^T\mathbf{u}_2 = 1$).

   (a) (10) True or False? $\left|\mathbf{u}_1^T\mathbf{u}_2\right| \leq 1$

   Mark one and explain.
   ▫ True ▫ False

(b) (10) True or False? If $\left|\mathbf{u}_1^T \mathbf{u}_2\right| = 1$, then $\mathbf{u}_1 = \pm\mathbf{u}_2$

Mark one and explain.

□ True □ False

(c) (10) Let $\mathbf{v}_1,\ \mathbf{v}_2$ be linearly independent vectors of magnitude 1. True or False? If $\mathbf{u}_i^T\mathbf{v}_j = 0$ for each i, j then the set $\{\mathbf{u}_1,\ \mathbf{u}_2,\ \mathbf{v}_1,\ \mathbf{v}_2\}$ is linearly independent.

Mark one and explain.

□ True □ False

2. (10) Let $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{u}_1$, $\mathbf{u}_2$ be two pairs of linearly independent vectors. True or False? $\mathbf{v}_1\mathbf{u}_1^T + \mathbf{v}_2\mathbf{u}_2^T$ has rank 2.

Mark one and explain.

□ True □ False

3. (10) Let A be an $n \times n$ matrix so that $A^T A = I$. True or False? $\det A^2 = 1$.

Mark one and explain.

□ True □ False

4. (20) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. If $\lambda_1 = 2$ and $\lambda_2 = 3$ are the eigenvalues of A compute $a_{11} + a_{22}$, and $\det A$.

$a_{11} + a_{22} =$
$\det A =$

5. (30) Let $\mathbf{v},\ \mathbf{w} \in \mathbf{R}^n$, and $a = \mathbf{v}^T\mathbf{w}$. Consider an $n \times n$ matrix $A = \mathbf{v}\mathbf{w}^T$.

(a) (10) Show that a and 0 are eigenvalues of A. Find an eigenvector $\mathbf{u}$ that corresponds to the eigenvalue a.

$\mathbf{u} =$

(b) (20) Find dimension $\dim V_a$ of the eigenspace that corresponds to the eigenvalue a, and dimension $\dim V_0$ of the eigenspace that corresponds to the eigenvalue 0.

$\dim V_a =$ $\qquad\qquad\qquad$ $\dim V_0 =$