MATH411
quiz 1
03/24/20
Total 100

By enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal.

Show all work legibly. **Name**:_____

1. (20) Let $\{p_1(z), \ldots, p_{n+1}(z)\}$ be elements of $\mathbf{P}_n$ so that $p_i(0) = 0$, $i = 1, \ldots, n+1$. True or False? The set $\{p_1(z), \ldots, p_{n+1}(z)\}$ is linearly independent.

Mark one and explain.

▫ True ▫ False

2. (20) Let $W = \{\mathbf{w}_1, \ldots, \mathbf{w}_n\}$, $n \geq 2$ be a vector set with at least one non zero vector. Denote by W_i a vector set with $n-1$ vectors obtained from W by removing $\mathbf{w}_i$. True or False? If $\mathbf{w}_i \in \text{span } W_i$ for each $i = 1, \ldots, n$, then dim span $W = 1$.

Mark one and explain.

▫ True ▫ False

3. (60) Let $W = \{\mathbf{w}_1, \ldots, \mathbf{w}_n\}$, $n \geq 2$ be a vector subset of a finite dimensional space V. If $i \in \{1, \ldots, n\}$ is the smallest index so that span $\{\mathbf{w}_1, \ldots, \mathbf{w}_{i-1}, \mathbf{w}_{i+1}, \ldots, \mathbf{w}_n\} = \text{span } W$, then denote $\{\mathbf{w}_1, \ldots, \mathbf{w}_{i-1}, \mathbf{w}_{i+1}, \ldots, \mathbf{w}_n\}$ by W^{-1}. If such an i does not exist, then define $W^{-1} = W$.

   (a) (15) True or False? If the vector set W is linearly independent, then $W^{-1} = W$.

Mark one and explain.

▫ True ▫ False

(b) (15) True or False? If $W^{-1} = W$, then the vector set W is linearly independent.

Mark one and explain.

▫ True ▫ False

(c) (15) For $k > 1$ define W^{-k} as $\left(W^{-(k-1)}\right)^{-1}$. True or False? If $W^{-k} = W^{-(k+1)}$, then $W^{-k} = W^{-(k+m)}$ for each $m \geq 1$.

Mark one and explain.

▫ True ▫ False

(d) (15) True or False? The vector set $\bigcap_{i=1}^{\infty} W^{-i}$ is linearly independent.

Mark one and explain.

▫ True ▫ False

4. (20) Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be linearly independent vectors. Suppose that $\mathbf{u}$ belongs to the span of each two vectors from $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Find $\mathbf{u}$ as a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

$\mathbf{u} =$