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Assignment 1:

(a) To find the point, $\mathbf{a_p}$, on L parameterized by $\mathbf{y} + t\,\mathbf{x}$, on which is projected the vector a, note that the vector $(\mathbf{y} - t_0\,\mathbf{x}) - \mathbf{a}$ is orthogonal to L and also the vector $\mathbf{x}$, at the point of projection, for some t_0, namely,

$$(\mathbf{y} + t_0\,\mathbf{x} - \mathbf{a})^T\,\mathbf{x} = 0 \;\Rightarrow\; \mathbf{y}^T\mathbf{x} + t_0\,\|\mathbf{x}\|^2 - \mathbf{a}^T\mathbf{x} = 0$$
$$\Rightarrow\; t_0 = (\mathbf{a}^T\mathbf{x} - \mathbf{y}^T\mathbf{x})/\,\|\mathbf{x}\|^2.$$

So, $\mathbf{a_p} = \mathbf{y} + t_0\,\mathbf{x}$, where $t_0 = (\mathbf{a}^T\mathbf{x} - \mathbf{y}^T\mathbf{x})/\,\|x\|^2$, which is the point that $\mathbf{a}$ projects on L. With this, $(\mathbf{a} - \mathbf{a_p})^T\mathbf{x} = 0$.

(b) Let $\mathbf{z} = \mathbf{y} + t_z\,\mathbf{x}$. Since $\mathbf{z}^T\mathbf{x} = 0$, then

$$(\mathbf{y} + t_z\,\mathbf{x})^T\,\mathbf{x} = 0 \;\Rightarrow\; \mathbf{y}^T\mathbf{x} + t_z\,\|\mathbf{x}\|^2 = 0$$
$$\Rightarrow\; t_z = -(\mathbf{y}^T\mathbf{x})/\,\|\mathbf{x}\|^2.$$

So, $\mathbf{z} = \mathbf{y} - [(\mathbf{y}^T\mathbf{x})/\,\|\mathbf{x}\|^2]\,\mathbf{x}$.

(c) Since $\mathbf{x}^T\mathbf{x} = 1$, then $\|\mathbf{x}\|^2 = 1$, and since $\mathbf{y}^T\mathbf{x} = 0$, then

$$\mathbf{a_p} = \mathbf{y} + [(\mathbf{a}^T\mathbf{x} - \mathbf{y}^T\mathbf{x})/\,\|\mathbf{x}\|^2]\,\mathbf{x}$$
$$= \mathbf{y} + (\mathbf{a}^T\mathbf{x})\,\mathbf{x}.$$