Assignment 4

1. Assignment 3, Problem 1a ii.

   You write: "$B^T\mathbf{x}$ is an $n \times n$ matrix." If B is an $n \times m$ matrix, and $\mathbf{x} \in \mathbf{R}^n$ (i.e. an $n \times 1$ matrix), then $B^T\mathbf{x}$ is an $m \times 1$ matrix, i.e. a vector in $\mathbf{R}^m$.

2. Assignment 3, Problem 1a iii.

   You write: "The dot product of a matrix with itself..." I am familiar with a dot product of two vectors, and agree $\mathbf{x}^T\mathbf{x} \geq 0$. I do not know what the dot product of two matrices is.

3. Assignment 3, Problem 1b.

   You are right by saying that the eigenvalues of a symmetric matrix with real entries are all real. The proof, however, is missing. Can you provide the proof? This becomes **Assignment 4, Problem 1**.

4. Assignment 3, Problem 2.

   You came up with the expression $\mathbf{a}^T\mathbf{a} - 2\mathbf{a}^T\mathbf{y} - \mathbf{x}^T\mathbf{a}\mathbf{a}^T\mathbf{x}$. I believe this expression equals to $|\mathbf{a}-\mathbf{y}|^2 - \left|\mathbf{x}^T\mathbf{a}\right|^2$ (am I right?). If so, then to identify the least squared approximation line (see Assignment 2, item 2) we would need to solve the following problem

   $$\min_{\mathbf{x},\mathbf{y}} \left\{ \sum_{i=1}^{m} |\mathbf{a}_i - \mathbf{y}|^2 - \left|\mathbf{x}^T\mathbf{a}_i\right|^2 \text{ subject to } \mathbf{x}^T\mathbf{x} = 1,\ \mathbf{x}^T\mathbf{y} = 0 \right\} \tag{1}$$

   (see Assignment 3, formula 2). The expression involves two unknown $\mathbf{x}$ and $\mathbf{y}$ and they are related ($\mathbf{x}^T\mathbf{y} = 0$). We assume first that $\mathbf{x}$ with $|\mathbf{x}| = 1$ is known, and will try to identify $\mathbf{y}$ so that

   $$\min_{\mathbf{y}} \left\{ \sum_{i=1}^{m} |\mathbf{a}_i - \mathbf{y}|^2 \text{ subject to } \mathbf{x}^T\mathbf{y} = 0 \right\} \tag{2}$$

   (if $\mathbf{x}$ is known, then $\sum_{i=1}^{m} \left|\mathbf{x}^T\mathbf{a}_i\right|^2$ is just a constant). This is **Assignment 4, Problem 2**.