Orthogonal problems

1. Let a be a vector in R^n, $L = y + tx$ a line in R^n
   a. Translate the coordinate space so the line passes through the origin, $L = tx, a = a - y$
   b. Let the orthogonal vector to L distance t_p on L be defined as $(a - y) - t_p x$
   c. Then $((a - y) - t_p x) \cdot x = 0$
      i. Solving for t_p:
         a. $(a - y) \cdot x - t_p x \cdot x = 0$
         b. $t_p = \frac{(a-y)\cdot x}{x \cdot x}$
   d. This gives the projection of $(a - y)$ onto the line tx: $\frac{(a-y)\cdot x}{x\cdot x} x$
   e. Translate back, so the final projection would be: $\frac{(a-y)\cdot x}{x\cdot x} x + y$

2. Let z be a vector in R^n.
   a. Since $z^T x = 0, z$ is orthogonal to x
   b. Find the vector that translates tx to the same location as y, but is orthogonal to x
      i. This should be the vector $y - t_p x$, with t_p such that $(y - t_p x) \cdot x = 0$
      ii. Solve for t_p:
         a. $(y \cdot x) - t_p(x \cdot x) = 0$
         b. $t_p = \frac{y\cdot x}{x\cdot x}$
   c. Thus the vector $z = y - \frac{y\cdot x}{x\cdot x} x$
   d. Note that unless y is also orthogonal to x, although the lines $y + tx$ and $z + tx$ are identical, for a given point the t values will not be the same unless y is also orthogonal to x.

3. From 1. the orthogonal projection should be $\frac{(a-y)\cdot x}{x\cdot x} x + y$ which can now be simplified:
      i. $\frac{a\cdot x - y\cdot x}{x\cdot x} x + y$
      ii. $\frac{a\cdot x - 0}{1} x + y$
      iii. $(a \cdot x)x + y$

The final vector would then be $(a \cdot x)x + y$