1. A) $\boldsymbol{x}$ is an $n \times 1$ matrix, and $\boldsymbol{y^T}$ is a $1 \times m$ matrix. Therefore the product is an $n \times m$ matrix.
    a. The product $\boldsymbol{xy^T}$ is the matrix $\begin{bmatrix} x_1y_1 & \cdots & x_1y_m \\ \vdots & \ddots & \vdots \\ x_ny_1 & \dots & x_ny_m \end{bmatrix}$
    b. This can be simplified to $[y_1\boldsymbol{x} \quad \dots \quad y_m\boldsymbol{x}]$ where $\boldsymbol{x}$ is the original $n \times 1$ column vector
    c. Since every column in the $\boldsymbol{xy^T}$ is a scalar multiple of the $\boldsymbol{x}$ column vector, they are not linearly independent. Since $\boldsymbol{x}$ and $\boldsymbol{y^T}$ are nonzero, at least one column is nonzero, so the rank is 1.

B) True. This can be shown by proving for any i,j $BB^T_{i,j} = A_{i,j}$ and BB^T and A are matrices of equivalent dimensions.

Since B is a $m \times n$ matrix, B^T is a $n \times m$ matrix. Therefore BB^T is a $m \times m$ matrix.
A = $\sum b_k b_k^T$ from $k = 1$ to n, where b_k is a $m \times 1$ matrix. Thus $b_k b_k^T$ is an $m \times m$ matrix.
Therefore A is the sum of $m \times m$ matrices so it is also a $m \times m$ matrix.

The $BB^T_{i,j}$ entry can be defined as $\sum B_{ik}B_{kj}^T$ from $k = 1$ to n. But $B_{kj}^T = B_{jk}$, so we can simplify this to $\sum B_{ik}B_{jk}$ from $k = 1$ to n.

The $A_{i,j}$ entry is i,j entries of $\sum b_k b_k^T$ from $k = 1$ to n. For a given k, the i,j entry of $b_k b_k^T$ is product of the i^{th} value of the k column of B, and the j^{th} value in the k column of B. So the $A_{i,j}$ entry can be formulated as $\sum B_{ik}B_{jk}$ from $k = 1$ to n.

Therefore BB^T and A are both $m \times m$ matrices whose i,j entries are equivalent, so BB^T = A.

C) True. Let C be the matrix BB^T. C is symmetric if and only if C = C^T.
$C^T = (BB^T)^T = (B^T)^T B^T = BB^T$.
Thus C = C^T so C is symmetric.

2. From problem 3 last week, if $L = t\boldsymbol{x} + \boldsymbol{y}$ has the properties $\boldsymbol{x}^T\boldsymbol{x} = 1$ and $y^T\boldsymbol{x} = 0$, the projection of a vector $\boldsymbol{a}_i$ onto L can be simplified to $(\boldsymbol{a}_i \cdot \boldsymbol{x})\boldsymbol{x} + \boldsymbol{y}$.
    a. The equation then becomes $\sum|\boldsymbol{a}_i - (\boldsymbol{a}_i \cdot \boldsymbol{x})\boldsymbol{x} - \boldsymbol{y}|^2$ from $i = 1$ to m