1. Problem 1
    a. Let B be an $m\ x\ n$ matrix, $\boldsymbol{x} \in R^n$. Show $\boldsymbol{x}^T BB^T \boldsymbol{x} \geq 0$
        i. $\boldsymbol{x}^T BB^T \boldsymbol{x} = (\boldsymbol{x}^T B)(B^T \boldsymbol{x}) = (B^T \boldsymbol{x})^T (B^T \boldsymbol{x}) = B^T \boldsymbol{x} \cdot B^T \boldsymbol{x}$
        ii. $B^T \boldsymbol{x}$ is an $n\ x\ n$ matrix.
        iii. The dot product of a matrix with itself is always positive, unless the matrix is zero then it is zero, so $\boldsymbol{x}^T BB^T \boldsymbol{x} \geq 0$.
    b. False: the eigenvalues of a symmetric matrix A are real only if the matrix A is real.
        i. Ex. Symmetric matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 2i \end{bmatrix}$
            1. The characteristic equation is $2i - 2i\lambda - \lambda + \lambda^2$
            2. Then the roots are $\lambda = 2i, 1$ so it has a non-real eigenvalue.
    c. True: Given a positive semi-definite matrix A, we have the property $\boldsymbol{x}^T A\boldsymbol{x} \geq 0$Q
        i. Let λ be an eigenvalue of A. Then $A\boldsymbol{x} = \lambda\boldsymbol{x}$.
        ii. Multiply both sides by $\boldsymbol{x}^T$: $\quad \boldsymbol{x}^T A\boldsymbol{x} = \boldsymbol{x}^T \lambda\boldsymbol{x}$.
        iii. Then from i. $\boldsymbol{x}^T \lambda\boldsymbol{x} \geq 0$
        iv. Since λ is a scalar $\lambda\boldsymbol{x}^T \boldsymbol{x} \geq 0$
        v. So finally $\lambda(\boldsymbol{x} \cdot \boldsymbol{x}) \geq 0$, and since $(\boldsymbol{x} \cdot \boldsymbol{x}) \geq 0, \lambda \geq 0$, so the eigenvalue is non-negative.

2. Problem 2: simplify $|a - xa^T x - y|^2$
    a. $\big((a-y) - xa^T x\big) \cdot \big((a-y) - xa^T x\big)$
    b. $(a-y)\cdot(a-y) - 2(a-y)\cdot(xa^T x) + (xa^T x)\cdot(xa^T x)$
    c. $a \cdot a - 2(a \cdot y) + y \cdot y - 2a \cdot (xa^T x) + 2y \cdot (xa^T x) + (xa^T x) \cdot (xa^T x)$
        i. Since all of these are column vectors or column vectors multiplied by scalars, the dot product equals the transpose of the vector multiplied by itself.
    d. $a^T a - 2a^T y + y^T y - 2a^T xa^T x + 2y^T xa^T x + (xa^T x)^T (xa^T x)$
    e. $a^T a - 2a^T y + y^T y - 2a^T xa^T x + 0 + (xa^T x)^T (xa^T x)$
        i. $(xa^T x)^T = \big((xa^T)x\big)^T = (x^T (xa^T)^T) = (x^T ax^T)$
    f. $a^T a - 2a^T y + y^T y - 2a^T xa^T x + (x^T ax^T)(xa^T x)$
    g. $a^T a - 2a^T y + y^T y - 2a^T xa^T x + x^T aa^T x$
        i. $2a^T xa^T x = 2x^T aa^T x$
    h. $a^T a - 2a^T y + y^T y - x^T aa^T x$
    Which should be a scalar