Assignment 5

4.1 That was the idea, $\overline{A}$ was obtained by substituting a_{ij} for $\overline{a}_{ij}$, so if A is real then $A = \overline{A}$

5.1 Assume $\boldsymbol{x} \neq 0$, then $\overline{\boldsymbol{x}} \neq 0, \overline{\boldsymbol{x}}^T \neq 0$.

$\overline{x}^T x = \overline{x}_1 x_1 + \overline{x}_2 x_2 + \cdots + \overline{x}_n x_n$ which is the sum of all positive nonzero numbers. $\overline{x}x = |x|^2 \rightarrow \overline{x}x > 0$.

Therefore $\overline{x}^T x > 0$.

5.2 $min\sum|a_i - z|^2 = \sum\sum\left(a_{ji} - z_i\right)^2$ from $j = 1$ to m and $i = 1$ to n.

1. Taking the partial derivative with respect to z_i gives $\sum -2(a_{ji} - z_i)$

   from $j = 1\ to\ m$ for the i[th] dimension.

2. Set the partial derivative equal to zero to find the minimum z_i

   $0 = \sum -2(a_{ji} - z_i)$

   $m * z_i = \sum a_{ji}$

   $z_i = \frac{\sum a_{ji}}{m}$ the minimum for 1 dimensions of z

3. Thus for all dimensions the minimum z can be simplified to $\frac{(a_1+a_2+\cdots+a_m)}{m}$

5.3 $min\sum a_i\left[\left(\frac{x}{a_i}\right)\log\left(\frac{x}{a_i}\right) - \left(\frac{x}{a_i}\right) + 1\right] = min\sum xlog\left(\frac{x}{a_i}\right) - x + a_i$

1. Taking the derivative with respect to x gives $\sum\log\left(\frac{x}{a_i}\right)$

2. Setting the derivative equation equal to zero gives a minimum when

   $$x = a_i, a_i \neq 0$$

3.

5.4 Due to the constraint $x^T y = 0$, any vector y that minimizes $\sum|a_i - y|^2$ must lie on the hyperplane $x^T y = 0$.

As shown in 5.2 the vector $c = \frac{a_1+a_2+\cdots+a_m}{m}$ solves the unconstrained minimization problem$\sum|a_i - y|^2$.

The orthogonal projection minimizes the distance between two vectors. Therefore the closest point on the hyperbolic plane $x^T y = 0$ to c is the orthogonal projection. So all that needs to be shown for this method to be true is the farther you get from the centroid, the 'bigger' the solution. This can be seen by looking at the second partial derivatives.

From 5.2, $min\sum|a_i - z|^2 = \sum\sum\left(a_{ji} - z_i\right)^2$ from $j = 1$ to m and $i = 1$ to n

The first partial with respect to z_i is $\sum -2(a_{ji} - z_i)$ from $j = 1\ to\ m$

The second partial with respect to z_i is just $2m$.

Therefore, the second derivatives are always positive, meaning the value of the solution set is always increasing as you move away from the minimum. Thus the minimal solution on the hyperbolic plane is one closest to the minimum, which is the orthogonal projection of c onto the plane.

6. New Problem

a. $|a - y|^2 - |x^T a|^2 = |a - c + xc^T x|^2 - |x^T a|^2 =$

$$|(a-c) + xc^T x|^2 - |x^T a_i|^2 = \left((a-c) + xc^T x\right) \cdot \left((a-c) + xc^T x\right) - (x^T a) \cdot (x^T a) =$$

$$(a-c)\cdot(a-c) + 2(a-c)\cdot(xc^T x) + (xc^T x)\cdot(xc^T x) - (x^T a)\cdot(x^T a) =$$

$$(a-c)\cdot(a-c) + 2a\cdot(xc^T x) - 2c\cdot(xc^T x) + (xc^T x)\cdot(xc^T x) - (x^T a)\cdot(x^T a) =$$

$$(a-c)\cdot(a-c) + 2a^T xc^T x - 2c^T xc^T x + (xc^T x)^T(xc^T x) - (x^T a)^T(x^T a) =$$

$$(a-c)\cdot(a-c) + 2a^T xc^T x - 2c^T xc^T x + x^T cx^T xc^T x - (x^T a)^T(x^T a) =$$

$$(a-c)\cdot(a-c) + 2a^T xc^T x - 2c^T xc^T x + c^T xc^T x - (x^T a)^T(x^T a) =$$

$$(a-c)\cdot(a-c) + 2x^T ac^T x - c^T xc^T x - (x^T a)^T(x^T a) =$$

$$(a-c)\cdot(a-c) - (-2x^T ac^T x + c^T xc^T x + (x^T a)^T(x^T a)) =$$

$$(a-c)\cdot(a-c) - ((c^T x)^T(c^T x) - 2(x^T a)^T(c^T x) + (x^T a)^T(x^T a)) =$$

$$(a-c)\cdot(a-c) - (c^T x - x^T a)\cdot(c^T x - x^T a) =$$

$$|a-c|^2 - |c^T x - x^T a|^2 =$$

$$|a-c|^2 - |c^T x - a^T x|^2 =$$

$$|a-c|^2 - |(c^T - a^T)x|^2$$

If this simplification is correct, the problem would become:

$$min\sum|a_i - c|^2 - \left|(c^T - a_i^T)x\right|^2 \text{ from } i = 1 \text{ to } m$$

b. minimizing this would be maximizing the dot product of $(c^T - a_i^T)$ with normalized vector x