Back to results

On Gevrey regularity of equations of fluid and geophysical fluid dynamics with applications to 2D and 3D turbulence

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Full text - PDF | Preview - PDF | Abstract/Details | References 75

Abstract Translate

The physical models of interest in this thesis are the Navier-Stokes equations (NSE) and surface quasi-geostrophic equation (SQG). We establish Gevrey regularity of solutions to these equations by combining Fourier analytic techniques with the semigroup approach of Weissler. This unifies several results regarding lower bound estimates on the radius of analyticity for the NSE, as well as provides an extension of the classical technique of Foias and Temam to so-called supercritical problems in the case of the SQG equation. Here and throughout, "criticality" refers to the discrepency in the order of dissipation and order of derivatives appearing in the nonlinearity, with *subcritical* specifically referring to the case where the order of dissipation dominates that of the nonlinearity, *critical* referring to the case where the orders are identical, and *supercritical* when the order of the nonlinearity is greater than the dissipation. This thesis can therefore be organized into two parts: Gevrey regularity of subcritical equations and Gevrey regularity of critical and supercritical equations.

In the first part of this thesis, we analyze a general, subcritical system, which includes as special cases, the NSE and subcritical SQG equation. We show that in the case of the NSE, we recover the best-known estimates for the maximal radius of spatial analyticity for both the two-dimensional (2D) and three-dimensional (3D) NSE in the context of turbulence. Moreover, our results suggest a path for potential improvement in the 3D case. Finally, we compare our method to the standard energy method and show that both approaches essentially give the same conclusions for the NSE, but that our method allows for an analysis of a more general system.

The second part of the thesis is dedicated to a particular supercritical problem, namely, the supercritical SQG equation. In this case, more care is needed when estimating the nonlinear term. In particular, the structure of the nonlinearity is exploited in a crucial way, in the form of a commutator, in order to ensure Gevrey regularity of solutions. In doing so, we present a method to extend the Gevrey norm technique of Foias and Temam to Besov spaces, as well as refine existing results concerning the regularity of solutions to the supercritical SQG equation in these spaces. We emphasize that the nature of Besov spaces and of the nonlinearity are exploited *together* in order to establish the desired estimates for the nonlinear term, for which we employ classical harmonic analysis techniques to derive.

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